## Dynamic Computational Networks

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February 18, 2016

## 1 What's the point?

Take a step back from linguistic analysis, and ask: what is the simplest way to perform the computations that are central and importans for the data of metrical systems?

From a cognitive point of view: What kind of [... neural...] hardware would be good at performing that kind of computation? Remember: the brain has no paper in it!

From a more traditional point of view, we may ask the question: why is there a strict distinction between the mechanism used for phonological representation and the mechanism used to modify the representations (rules, constraints, etc.)? Let's build a model in which the two are integrated.

## 2 Dynamic computational model



5 parameters:

1. $\alpha$ to the left
2. $\beta$ to the right
3. I Initial positional activation
4. F Final positional activation
5. $P$ Penultimate positional activation


- $a_{i}^{t}$ is the activation of the $i^{t h}$ unit at time $t$.
- Inherent activation: $\operatorname{Inh}(\mathrm{i})=\delta(1, i) \times I+\delta(-2, i) \times P+\delta(-1, i) \times$ where ${ }^{1}$

$$
\begin{array}{ll}
\delta(1, i)=1 & \text { iff } \mathrm{i}=1 \\
\delta(-2, i)=1 & \text { iff } \mathrm{i} \text { is the penultimate position; } \\
\delta(-1, i)=1 & \text { iff } \mathrm{i} \text { is the ultimate position. }
\end{array}
$$

[^0]$$
a_{i}^{t}=\operatorname{Inh}(i)+\alpha \times a_{i+1}^{t-1}+\beta \times a_{i-1}^{t-1}
$$

\[

$$
\begin{aligned}
& - \text { Ist sylloble } \longrightarrow-2 \text { nd sylleble } \rightarrow *-3 \text { rd sylloble } \\
& -\boxminus-4 \text { ih sylable } \rightarrow-5 \text { h syllable }-5 \text { th syllable }
\end{aligned}
$$
\]

## 3 Sonority and syllabification

### 3.1 Tashlhit Berber

Dell and Elmedlaoui

| 8 | 7 | 5 | 4 | 3 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| a | i u | liquids | nasals | voiced fric | voiceless fric | voiced stops | voiceless stops 9.


| t | l | w | a | t | $(\alpha, \beta)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| o | 5 | 7 | 8 | o | inherent activation |  |  |  |  |
| -1.1 | 3.6 | 4.6 | 8.0 | 0 |  | $(-0.3,0)$ |  |  |  |
| -1.8 | 3.5 | 3.0 | 8.0 | 0 | $(-0.5,0)$ |  |  |  |  |
| -2.1 | 4.2 | 2.2 | 8.35 | 1.3 | $(-0.5, .15)$ |  |  |  |  |
| -2.6 | 4.3 | 1.5 | 8.4 | -0.9 | $(-0.6, .1)$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| t | i | z | r | w | a | 1 | i | n | $(\alpha, \beta)$ |
| o | 7 | 3 | 5 | 7 | 8 | 5 | 7 | 4 | inherent activation |
| -4.4 | 7.3 | 0.2 | 3.4 | 2.6 | 6.8 | 1.6 | 4.6 | 3.8 | $(-0.6,-0.1)$ |

This next word is especially interesting, because it illustrates how a high sonority segment can fail to be a syllable nucleus, because its sonority is dampened by its right-hand neighbor:

| i | h | a | u | l | t | n | $(\alpha, \beta)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 2 | 8 | 7 | 5 | 0 | 4 | inherent activation |
| 8.85 | -3.1 | $\mathbf{7 . 0}$ | 2.2 | 6.9 | -3.5 | 4.6 | $($ (-o.6,-o.1 $)$ |

## 4 Stress and cyclicity: Indonesian

Based on material in Cohen 1989.

| bicára | speak |
| :--- | :--- |
| bìjiksána | wise |
| xàtulistíwa | equator |
| òtobìográfi | autobiography |
| àmerikànisási | Americanization |

Cohen's analysis:

1. Final syllable is extrametrical.
2. End rule: Final ("penultimate stress").
3. End Rule: Initial (but blocked if clash would ensue).
4. Perfect Grid (Right to Left, blocked if clash would ensue)

| rule | O O O | O OOO | O O O O O | OOOOOO |
| :---: | :---: | :---: | :---: | :---: |
| 1 | O o (o) | O o o (o) | O o o o (o) | o o o o o (o) |
| 2 | o ó (o) | o o ó (o) | o o o ó (o) | o o o o ó (o) |
| 3 | clash | ó o ó (o) | ó o o ó (o) | ó o o o ó (o) |
| 3 | o ó (o) | ó o ó (o) | ó o o ó (o) | ó o ó o ó (o) |
| $\alpha=-0.5$ | $\beta=0.0$ | $\mathrm{I}=0.7$ | $\mathrm{P}=1.0$ |  |
|  | 0.710 | 0.7010 | 0.70010 | 0.700010 |
|  | 0.21 .00 .0 | 0.95 -0.5 1.0 0.0 | 0.58 0.25-0.5 1.0 0.0 | $0.76-0.130 .25-0.51 .00 .0$ |

Morphologically complex cases:

| 1st cycle | o o o o o o |
| :--- | :--- |
| 1 | o o o o (o) |
| 2 | o o o ó (o) |
| 3 | ó o o ó (o) |
| 4 | DNA (clash avoidance) |
| 2 nd cycle | ó o o ó ó o |
| 1 | ó o o ó o (o) |
| 2 | ó o o ó ó (o) |
| 3 | ó o o ó ó (o) |
| 4 | DNA (clash avoidance) |
| output <br> clash resolution <br> output | ó o o o ó ó ó o o o (o) |


| $\alpha=-0.5$ | $\beta=0.0$ | $\mathrm{I}=0.7$ | $\mathrm{P}=1.0$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.7 | 0 | 0 | 1 | 1 | 0 |
|  | $\mathbf{0 . 6 4}$ | 0.13 | -0.25 | -0.5 | $\mathbf{1 . 0}$ | 0.0 |

## 5 Stress systems: typology

Examples, from Hayes

### 5.1 Pintupi

(Hansen and Hansen 1969, 1978; Australia):
"syllable trochees": odd-numbered syllables (rightward); extrametrical ultima:

| S | s |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S | s | s |  |  |  |  |
| S | s | S | s |  |  |  |
| S | s | S | s | s |  |  |
| S | s | S | s | S | s |  |
| S | s | S | s | S | s | s |

### 5.2 Weri

Boxwell and Boxwell 1966, Hayes 1980, HV 1987

| Øintíp | bee |
| :--- | :--- |
| kùlipú | hair of arm |
| ulùamát | mist |
| àkunètepál | times |

- Stress the ultima, plus
- Stress all odd numbered syllables, counting from the end of the word.

$\mathrm{I}=0.0$
$\mathrm{F}=1.0$
$\alpha=-0.8$
$\beta=0.0$


### 5.3 Warao

(Osborn 1966, HV 1987)
Wercoo


$\mathrm{I}=0.0$
$\mathrm{F}=-1.0$
$\alpha=-0.7$
$\beta=0.0$

- Stress penult syllable;
- all even-numbered syllables, counting from the end of the word.
jiwàranáe he finished it
japurùkitàneháse verily to climb
enàhoròahàkutái the one who caused him to eat


### 5.4 Maranungku

${ }^{2}$ Stress first syllable, and All odd-numbered syllables from the
beginning of the word.

$$
\begin{aligned}
& \mathrm{I}=1.0 \\
& \mathrm{~F}=0.0 \\
& \alpha=0.0 \\
& \beta=-0.7
\end{aligned}
$$

3 ... or Indonesian, ...


Garawa (Furby 1974) illustrates this quite common class: accent falls on both the initial syllable and on the penult, corresponding to a positive setting of I , a negative setting of F , and a negative value of $\alpha$ (in order that the negative value of F should translate into a positive value for the penultimate syllable). In such systems, we typically find either accent iterating from left to right, on oddnumbered syllables counting from the first, or else accent iterating from right to left, on every other syllable to the left of the penult, depending on the relative magnitudes of $\alpha$ and $\beta$. Garawa falls into the latter category, and this pattern illustrates the result of a system in which the $\alpha$-effect is stronger than the $\beta$ effect: in which, that is, $\alpha<\beta$ (though, more to the point, the absolute value, $|\alpha|$ is greater than $|\beta|$, since $\alpha$ is negative)

| yámi | eye |
| :--- | :--- |
| púnjala | white |
| wátjimpàyu | armpit |
| nárininmùkunjìnamìra | at your own many |

- Stress on Initial syllable;
- Stress on all even-numbered syllables, counting leftward from the end; but "Initial dactyl effect": no stress on the second syllable permitted.
$\mathrm{I}=1.0$
$\mathrm{F}=-0.5$
$\alpha=-0.7$
$\beta=-0.1$


### 5.6 Lenakel

Lynch 1978; Hayes 1995: 167-78. As is well-known, accent in Lenakel is unusual in that stress is assigned according to principles that appear to be quite different in nouns when compared with the principles operative in verbs and adjectives. Verbs and adjectives (see ) are stressed on the penultimate syllable, on the first syllable, and on every alternate (odd numbered) syllable as we count from left to right, starting with the beginning of the word, with the exception
that the antepenult is never stressed. Nouns, on the other hand, bear penultimate stress, and show a pattern of accent assignment on alternate syllables counting from the end of the word, alternating leftward from the penultimate syllable.

I = 1.0 for verbs and adjectives; o.o for nouns
$\mathrm{F}=-0.5$
$\alpha=-0.4$
$\beta=-0.6$
verbs and adjectives:

| r`iməlg' \(\varepsilon y g \varepsilon y\) & he liked it \\ n`imarəlg' $\varepsilon g \varepsilon y$ | you p. liked it |
| :--- | :--- |
| n`imamàrolg' \(\varepsilon y g \varepsilon y\) & you pl. were liking it \\ t`inagàmarəlg' $\varepsilon y g \varepsilon y$ | you pl. will be liking it |
| nouns (four or more syllables): |  |
| nimwàgəlágəl <br> tubwàlugálvkh | beach |




This pattern is a peculiar embarrassment to traditional accounts of Lenakel, accounts which distinguish essentially between rules and representations. In nouns, not only is the initial stress of the verbs missing, but the direction of iteration of the rule that creates alternating stress must change depending on lexical category. In the present model, however, nothing of the kind is necessary; not only is this case not an embarrassment, it is precisely the kind of case
that is predicted by the theoretical model. We need simply say that in the case of nouns, there is no Initial activation; crucially, however, the values of $\alpha$ and $\beta$ remain fixed across the entire language. Because there is no Initial activation in the case of nouns, there is no rightward-spreading wave for the $\beta$-coefficient to pass on. There is, from a mathematical point of view, both a wave propagated leftward and a wave propagated rightward; the one which is stronger will, by and large, drown out the other from a purely quantitative point of view, but when the rightward moving wave is removed, by the non-occurrence of initial stress in the nominal system, the wave moving sotto voce leftward from the penult becomes entirely audible.

## 6 Quantity-insensitive systems with bias

We have so far considered only cases where the internal activations to all segments was zero, equally and across the board. By definition, quantity-insensitive systems assign equal internal activation to each unit, but that activation need not be zero; it may be a quantity (which we shall refer to as bias) which all the units uniformly receive. A non-zero bias will give rise to a rhythmic system as well, where a negative bias is applied, and in, where a positive bias is assigned. Rhythmicity of much the sort that we have already explored is inherent to the system, whether activation comes in from one unit or from all of them.i In these two examples, the rhythm emerges in a $\beta$-dominant system, i.e., one where $\beta$ is significantly negative and $\alpha$ is zero or negligibly close to it.

Negative bias:

$$
\begin{aligned}
& \mathrm{I}=0.0 \\
& \mathrm{~F}=0.0 \\
& \alpha=0.0 \\
& \beta=-0.8 \\
& \text { bias }=-1.0
\end{aligned}
$$



Positive bias:
$\mathrm{I}=0.0$
$\mathrm{F}=0.0$

$$
\begin{aligned}
& \alpha=0.0 \\
& \beta=-0.8
\end{aligned}
$$

$$
\text { bias }=1.0
$$



Positive bias:
$\mathrm{I}=0.0$
$\mathrm{F}=-1.2$
$\alpha=0.0$
$\beta=-0.8$
bias $=-1.0$


## 7 Quantity sensitive systems

Consider the contrast between two similar metrical systems (following here a discussion in van der Hulst (ms.)): both Rotuman (Churchward 1940) and Yapese (Jensen 1977) are quantity-sensitive systems in which stress falls on the ultima or the penult, depending on syllable weight. If, in that final window of two syllables, there is only one heavy syllable, then that syllable is the stressed syllable. If there are two heavy syllables (i.e., if both the penult and the ultima are heavy), then the final syllable is stressed. The systems differ, however, with respect to where stress falls when both the ultima and the penult are light: in Rotuman, the stress falls on the penult, and in Yapese, the stress falls on the ultima.

|  | Language | H L ] | L H ] | L L ] | H H ] |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | Rotuman | ó o | o ó | ó o | o ó |
|  | Yapese | ó o | o ó | o ó | o ó |
| bias |  |  |  |  |  |
| negative <br> positive | Rotuman | $\mathbf{1 . 8 - 1 . 0}$ | $\mathbf{- 1 . 8} \mathbf{1 . 0}$ | $\mathbf{- 0 . 2 - 1 . 0}$ | .2 1.0 |

## 8 Lateral inhibition

In a 1- or 2-dimensional array of neurons, neurons:

- excite very close neighbors;
- inhibit neighbors in a wider neighborhood;
- do not affect cells further away

This can serve to create edge detectors.
Consider the following diagram, in which the lower line represents a 1-dimensional retina, and each spot it assigned a number, and in which the upper line represents the difference between the activation of each unit. The lower diagram represents the function $f$, and the higher diagram represents $\Delta \mathrm{f}$, where $\Delta \mathrm{f}(\mathrm{i})=\mathrm{f}(\mathrm{i})-\mathrm{f}(\mathrm{i}-\mathrm{I})$.


1
0
123456789
In the next diagram, the function on top is

$$
\Delta^{2} f(i)=2 f(i)-f(i-1)-f(i+1)
$$

although it is better to think of it as

$$
\Delta^{2} f(i)=[f(i)-f(i-1)]-[f(i+1)-f(i)]
$$

which is the difference of two successive differences. That makes this the discrete equivalent of the second derivative. 4 A unit on the upper tier is (doubly) activated by its own corresponding element on the lower tier, and inhibited by the neighbor on its left and neighbor on its right. The total effect is to have no activation on this tier in a field of constant activation, and to have an exaggerated response to changes in activation, identifying "edges", areas where activation is changing.
${ }^{4}$ If we were doing this in more than one dimension, we would appeal to the laplacian, which is the sum of the second derivatives around a point.



[^0]:    ${ }^{1}$ This is a pretty commonly used notational trick to simplify the algebra, sometimes written $\delta_{j}^{i}$.

